BIHAR MATHEMAICAL SOCIETY

TSTM_JUNIOR Model Questions Full marks: 100

Full time: 2:30 hours

Write all Questions Answer in your own words, Each question has same marks

1. Determine the smallest positive integer x, whose last digit is 6 and if we erase last digit 6 and put it in the left most of the number the new number becomes $4x$.
2. Let a, b, c be the length of the sides of a triangle and x be the in-radius. Show that
$$x < \frac{a^2 + b^2 + c^2}{3(a + b + c)}$$
3. P is randomly chosen point in a unit square. Connect p to the vertices of the base. Find the probability that the triangle is acute angled.
4. Find the number of numbers $\leq 10^8$ which are neither perfect squares, nor perfect cubes, nor perfect fifth powers.
5. m and n are two positive integers such that $m = n^2 - n$. show that $m^2 - m$ is divisible by 24.
6. Let ABCD be a parallelogram. Two points E and F are chosen on the sides BC and CO, respectively, such that $\frac{EB}{EC} = m$, and $\frac{FC}{FD} = n$, lines AE and BF intersect at G. Prove that the ratio
$$\frac{AG}{GE} = \frac{(m+1)(n+1)}{mn}$$
7. One root of the equation $x^4 - 5x^3 + ax^2 + bx + c = 0$ is $3+\sqrt{2}$. If all the roots of the equation are real then find the extreme (maximum and minimum) values of a, b and c and the roots of the equation for these extreme values of a, b and c. It is given that a, b, and c are rational.
8. Consider the collection of all 5-digits of each number being 43. A number is selected at random from the collection. Find the probability that the number is divisible by 11.
9. Evaluate $\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{10000}}\right]$, where $[x]$ is greatest integer $\leq x$.
10. A square sheet of paper ABCD is so folded that B falls on the midpoint m of CD. Prove that the crease will divide BC in the ratio $5 : 3$.